Complexity and Energy Trade-off in Quantum Protocols

Yukuan Tao, Madalin Guta, Gerardo Adesso
University of Nottingham

In this work, we present a novel perspective to analyse the relation between channel complexity and total energy cost of quantum protocols. In particular, we study a sequential quantum phase estimation protocol, where a phase of physical significance is encoded in a quantum channel on an optical platform. The channel is applied to a probe state repetitively until the probe is measured and the outcome leads to an estimate on the phase. We establish a trade-off relation between the total energy cost of the protocol and the number of times the channel is applied (query complexity), while reaching a desired estimation precision. The two fundamental quantities, often used as resource quantifiers, are linked through their dependence on the implementation precision of each phase-encoding channel. Our main finding is the existence of a finite precision level for complexity-energy co-optimisation, beyond which the reduction in one factor is overpowered by the overhead in the other. The location of such a sweet spot provides a recipe to identify optimal working points for quantum realizations that aim to achieve a desired target performance while keeping both gate complexity and energy cost at bay.

We anticipate the trade-off framework can be generalised to a wealth of more complex protocols, including all common quantum information processing and computation tasks for which a notion of complexity can be clearly identified. Our results reveal that an energy-efficient task may not need perfect implementation and a finite experimental error can in fact be beneficial. In this respect, the line of work initiated in this work has the potential to lead to general guiding principles for energy-green quantum technologies.

[1] Tao, Y., Gută, M., & Adesso, G. (2025). Trade-off between complexity and energy in quantum phase estimation.