Theoretical Lower Bounds on the Energetic Consumption of Continuous-Variable Quantum Key Distribution

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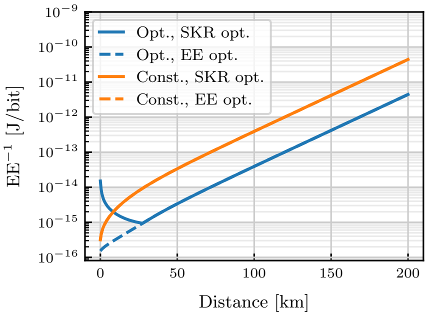
Quantum Key Distribution (QKD) is one of the most mature applications of quantum principles to technological purposes. First introduced in 1984, many protocols were proposed and implemented, both in laboratory conditions and on deployed fields, in the last 40 years, reaching such a level of maturity that commercial systems are available nowadays. However, the question of the resource cost and especially energetic consumption, essential in a world with finite resources and for a field that approaches practical deployment, has only been considered recently [1]. This question is however essential to compare different QKD protocols together, and optimize network architectures to minimize the energetic impact of such systems.

In [1], the notion of energy efficiency EE, known in the classical realm, was extended to quantum communication. In the context of point-to-point QKD, this quantity is expressed as the ratio of the secret key rate K to the instantaneous system power P; *i.e.* $\mathrm{EE} = K\ [\mathrm{bit/s}]/P\ [\mathrm{J/s}]$. It represents the number of secret key bits that can be extracted with 1 J of energy. The invert represents the amount of energy required to extract one bit. In [1], P was computed using a hardware dependent approach, considering the different components that compose the systems, giving relevant numbers for current and near-future systems.

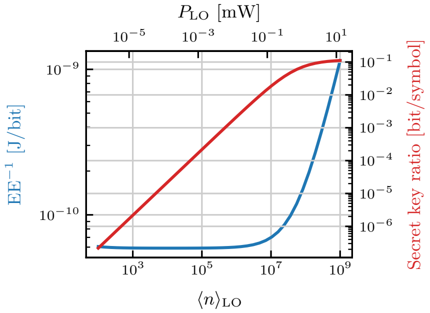
In this work, it is proposed to derive P as a theoretical minimum in the case of the Gaussian-Modulated Coherent State (GMCS) Continuous-Variable (CV) QKD protocol. The secret key rate K can be computed with standard techniques [2], as a function of the average photon number per symbol $\langle n\rangle$, the transmittance T, the excess noise ξ, the detection electronic noise V_{el}, detection efficiency η and the repetition rate of the system R. The instantaneous system power P is separated into the transmitter power P_{tx} and the receiver power P_{rx}, considering only the minimal energy to generate the states and perform the interferometric measurement. P_{tx} is computed using R and $\langle n\rangle$. P_{rx} appears, at first, more challenging to compute, as another beam, the Local Oscillator (LO), is used for the interferometric measurement. The work of [3] is used to constrain the number of photons in the LO, $\langle n\rangle_{\mathrm{LO}}$, as a function of T, η and $\langle n\rangle$ to form a valid heterodyne detection. Overall, the energy efficiency is expressed as a function of $\langle n\rangle$, T, ξ, V_{el}, η and R. As K and P are dependent on the crucial parameter $\langle n\rangle$, two cases are considered : the value that yields the highest energy efficiency EE, and the value that yields the highest key rate K.

Considering the perfect detector ($\eta = 1, V_{\mathrm{el}} = 0$), the analysis reveals the fundamental scaling of the energetic cost of CV-QKD, as shown in Fig. a. Considering a realistic detector with standard values for the electronic noise and efficiency, one finds a surprising result : the optimal EE is obtained in a scenario not normally considered for CV-QKD, one where the detector is not shot noise limited and where the electronic noise dominates (see Fig. b for an example a specific distance). Interestingly, while the EE spans one order of magnitude, the secret key rate spans more than 5 orders of magnitude, making the system way more efficient at only a small increased cost. While this shows the results at a specific distance, the behavior is confirmed as shown in Fig. c. The results were extended by considering the finite-size corrections, revealing the same scalings and findings.

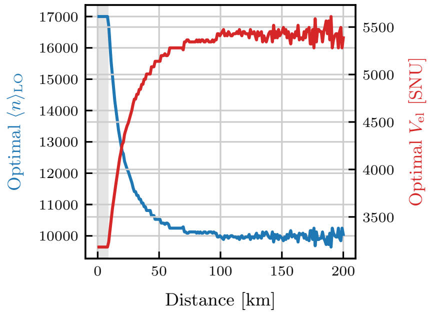
To conclude, this work provides the first theoretical analysis on lower bounds of the cost of CV-QKD, revealing the fundamental scaling of the energy cost of such protocols but also non-trivial counter-intuitive results for the realistic case.



(a) Invert of the energy efficiency vs distance for the perfect detector.



(b) Invert of the energy efficiency and secret key rate vs LO power for the realistic detector.



(c) Optimal $\langle n\rangle_{\mathrm{LO}}$ and optimal V_{el} as function of the distance.

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